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Cylindrically Symmetric Bulk Viscous Fluid in Bimetric Relativity

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Abstract

In this paper, cylindrically symmetric space-time is studied with bulk viscous fluid in the context of Rosen's Bimetric Theory of Relativity. Here it is shown that only vacuum model can be constructed.

Keywords: -Cylindrically symmetric, bulk viscous fluid, Bimetric Relativity

AMS Code-83C05 (General relativity).

Introduction

The bimetric theory of gravitation was proposed by Rosen[1][2](1940,1973) to modify the Einstein's general theory of relativity,by assuming two metric tensors.

In this theory he has proposed a new formulation of the general relativity by introducing a background Euclidean metric tensor γ_{ij} in addition to the usual Riemannian metric tensor g_{ij} at each point of the four dimensional space time. With the flat background metric γ_{ij} the physical content of the theory is the same as that of the general relativity.

Thus, now the corresponding two line elements in a coordinate system x^i are –

$$ds^2 = g_{ij}dx^i dx^j \qquad (1.1)$$

And

$$d\sigma^2 = \gamma_{ij}dx^i dx^j \qquad (1.2)$$

Where ds is the interval between two neighboring events as measured by means of a clock and a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable. One can regard it as describing the geometry that would exist if no matter were present.

Cylindrically symmetric perfect fluid distributions both static and non static have been discussed by many investigators for various matters in the context of general relativity. Heckman,O.,and Schucking,E.[3],Mardar[4],Benergee,A.,and Benergee,S.[5],Krori,K.D.and Barua,J.[6],Roy and Narain [7][8],H.Baysal [9] have studied various aspects of Cylindrically symmetric space-times. In addition to this cylindrically symmetric space-time is studied in bimetric relativity by Deo [10] with the matter cosmic strings and domain wall. In continuation of this study here we have investigated cylindrically symmetric space-time with bulk viscous fluid in bimetric relativity.

Field equations in bimetric relativity

Rosen N. has proposed the field equations of Bimetric Relativity from variation principle as

$$K_i^j = N_i^j - \frac{1}{2} N g_i^j = -8\pi\kappa T_i^j \qquad (2.1)$$

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Where $N_i^j = \frac{1}{2}\gamma^{\alpha\beta}\left[g^{hj}g_{hi|\alpha}\right]_{|\beta}$ (2.2)

$N = N_\alpha^\alpha$, $\kappa = \sqrt{\frac{g}{\gamma}}$ (2.3)

and $g = |g_{ij}|$, $\gamma = |\gamma_{ij}|$ (2.4)

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij}

3. Cylindrically symmetric space time with bulk viscous fluid:

We consider here the spherically symmetric line element of the form

$$ds^2 = A^2\left(dx^2 - dt^2\right) + B^2dy^2 + C^2dz^2 \tag{3.1}$$

Where A, B and C are functions of x and t only.

Corresponding to equation (3.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{3.2}$$

Since γ_{ij} is the Lorentz metric i.e. (1,1,1,-1), therefore γ- covariant derivative becomes the ordinary partial derivative.

And , T_i^j the energy momentum tensor for Bulk viscous fluid is given by

$$T_i^j = \left(\rho + \bar{p}\right)v_i v^j + \bar{p}g_i^j \tag{3.3}$$

Where $\bar{p} = p - \xi v^j{}_{;i}$ together with $g_{ij}v^i v^j = -1$ ie $v_4 v^4 = -1$ where v_i is the four-velocity of the bulk viscous fluid $\rho, p, \bar{p}$ and ξ are the energy density, isotropic pressure, effective pressure and bulk viscous coefficient.

In co-moving coordinate system we have

$T_1^1 = T_2^2 = T_3^3 = \bar{p}$, $T_4^4 = -\rho$ and $T_i^j = 0$ for $i \neq j$

Using equations (2.1) to (2.4) with (3.1) and (3.3),

We get the field equations as

$$\frac{1}{2}\left[\left(\frac{B'^2}{B^2} - \frac{B''}{B}\right) - \left(\frac{\dot{B}^2}{B^2} - \frac{\ddot{B}}{B}\right) + \left(\frac{C'^2}{C^2} - \frac{C''}{C}\right) - \left(\frac{\dot{C}^2}{C^2} - \frac{\ddot{C}}{C}\right)\right] = -8\pi\kappa\bar{p} \tag{3.4}$$

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$$\left(\frac{A'^2}{A^2}-\frac{A''}{A}\right)-\left(\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}\right)-\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)+\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=-8\pi\kappa\bar{p}$$

(3.5)

$$\left(\frac{A'^2}{A^2}-\frac{A''}{A}\right)-\left(\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}\right)+\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)-\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)+\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=-8\pi\kappa\bar{p}$$

(3.6)

$$\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)+\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=8\pi\kappa\rho$$

(3.7)

Where $A'=\frac{\partial A}{\partial x}, A''=\frac{\partial^2 A}{\partial x^2}, \dot{A}=\frac{\partial A}{\partial t}, \ddot{A}=\frac{\partial^2 A}{\partial t^2} etc$

In cylindrically symmetric inhomogeneous universe,the rotation ω^2 is identically zero,the expansion θ ,Sheer scalar σ^2 ,acceleration vector $\dot{v}_i$ and proper volume V^3 are found respectively to have following expressions-

$$\theta = v^i;_i = A^{-1}\left(\frac{A'}{A}+\frac{B'}{B}+\frac{C'}{C}\right) \quad (3.8)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\theta^2 - A^{-2}\left(\frac{A'B'}{AB}+\frac{A'C'}{AC}+\frac{B'C'}{BC}\right) \quad (3.9)$$

Where $\sigma_{ij} = \frac{1}{2}\left(v_{i;j}+v_{j;i}\right)-\frac{1}{3}\theta\left(g_{ij}-v_iv_j\right)$ (3.10)

$$\dot{v}_i = v_{i;j}v^j = \left(\frac{A'}{A},0,0,0\right) \quad (3.11)$$

Also $V^3 = \sqrt{-g} = A^2BC$ (3.12)

Using the Einstein's equations of general relativity we have-

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi\kappa T_{ij} \quad (3.13)$$

For the metric (3.1) with the equations (3.8) and (3.9) one may obtain the Raychoudhuri's equation as-

$$\theta = v^i;_i - \frac{1}{3}\theta^2 - 2\sigma^2 - \frac{1}{2}\rho_p \quad (3.14)$$

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Where $R_{ij}v^i v^j = \frac{1}{2}\rho_p$ (3.15)

Using equation (3.4) to (3.7), we get

$$\bar{p} + \rho = 0 \tag{3.16}$$

This equation of state is known as false vacuum.In view of reality conditions $\bar{p} > 0, \rho > 0$

Equation (3.16) immediately implies that $\bar{p} = 0, \rho = 0$ (3.17)

that is perfect fluid does not exists in cylindrically symmetric space-time in bimetric relativity.

On solving (3.4)-(3.7) with the help of (3.17)

We obtain

$$\left(\frac{A'^2}{A^2} - \frac{A''}{A}\right) - \left(\frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A}\right) = 0 \tag{3.18}$$

$$\left(\frac{B'^2}{B^2} - \frac{B''}{B}\right) - \left(\frac{\dot{B}^2}{B^2} - \frac{\ddot{B}}{B}\right) = 0 \tag{3.19}$$

$$\left(\frac{C'^2}{C^2} - \frac{C''}{C}\right) - \left(\frac{\dot{C}^2}{C^2} - \frac{\ddot{C}}{C}\right) = 0 \tag{3.20}$$

By using Method of separation of Variables, (3.18)-(3.20) gives us the solution

$$A = e^{\left[\frac{l}{2}\left(x^2+t^2\right)+l_1x+l_2t\right]} \tag{3.21}$$

$$B = e^{\left[\frac{l}{2}\left(x^2+t^2\right)+l_3x+l_4t\right]} \tag{3.22}$$

$$C = e^{\left[\frac{l}{2}\left(x^2+t^2\right)+l_5x+l_6t\right]} \tag{3.23}$$

where, $l, l_1, l_2, l_3, l_4, l_5$ and l_6 are the constants of integration.

Thus substituting the value of A, B and C in (3.1), we get the vacuum line element as

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$$ds^2 = \mathbf{exp}\,2\left[\frac{l}{2}\left(x^2+t^2\right)+l_1x+l_2t\right]\left(dx^2-dt^2\right)$$
$$+\mathbf{exp}\,2\left[\frac{l}{2}\left(x^2+t^2\right)+l_3x+l_4t\right]dy^2+\mathbf{exp}\,2\left[\frac{l}{2}\left(x^2+t^2\right)+l_5x+l_6t\right]dz^2 \quad (3.24)$$

Conclusion

In cylindrically symmetric , there is nil contribution of bulk viscous fluid in Bimetric theory of relativity respectively. It is observed that the matter fields like bulk viscous fluid cannot be a source of gravitational field in the Rosen's bimetric theory but only vacuum model exists.

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References

1. *Rosen N. (1940) General Relativity and Flat space I. Phys. Rev.57, 147.*
2. *Rosen N. (1973) A bimetric theory of gravitation I Gen. Rela. Grav. 04, 435-47.*
3. *Heckmann,O.,Schucking E.(1962)Gravitation,An introduction to current Research.John Wiley,New York,P.43*
4. *Mardar (1958) Gravitational waves in general relativity II The reflection of cylindrical waves.Proc.R.Soc.,246,133*
5. *Benergee A.,Benergee S.(1968) Stationary distribution of dust and Electromagnetic fields in general relativity.J.Phys.A.Ser.2,1,188.*
6. *Krori K.D.,Barua J.(1974) Magnetic energy and cylindrically symmetric distribution of matter in equilibrium in general relativity. Indian J.Pure Appyl.Phy.,12,818-22*
7. *Roy S.R. and Narain S.(1979)Some plane symmetric models of perfect fluid distribution in general relativity . Indian J.Pure Appyl.Phy.,10,763*
8. *Roy S.R. and Narain S.(1981) Cylindrically symmetric non-static perfect fluid distribution in general relativity with pressure equal to density. Indian J.Pure Appyl.Phy.,20,709*
9. *Baysal H(2001). Strings cosmological models in cylindrically symmetric inhomogeneous universe in genera relativity. Turk.J.Phys. 25, 283*
10. *Deo S.D.(2011) Cylindrically symmetric inhomogeneous cosmic strings and domain walls in bimetric relativity International journal of mathematical archive-2(1),page: 121- 123*